A FURTHER STUDY OF CENTIMETER SCALE "A."

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This scale has already been so fully described in the printed Proceedings of this Society, that further description is unnecessary.

It will be seen from the published reports of Professor Rogers and of Professor Hilgard, the Superintendent of the Bureau of Weights and Measures, that there is an outstanding difference in the results obtained amounting to about 1.4 mikrons. An effort has been made in the observations here recorded to ascertain which of the two results is correct.

The comparator used in making the comparisons described in this paper was made by Prof. W. A. Rogers, and consists essentially of a carriage carrying a microscope with a filar micrometer, and moving by means of a rack and pinion, between two stops, which limit the motion of the carriage approximately to 1 cm. The scales to be compared lie upon a table below the microscope, the table being capable of adjustment in two directions, one parallel to the line of motion of the carriage, and the other at right angles thereto, by which means each scale can be brought under the microscope used in making the comparisons. This table is furnished with an eccentric underneath one end, by means of which the scales to be compared can be readily and accurately brought into focus.

Centimeter "A" was first compared with a standard centimeter ruled on speculum metal by Prof. W. A. Rogers, and designated "B." The comparisons were continued from January 12 to March 21, 1886, at temperatures ranging from 34° F. to 70° F. The results of these comparisons, reduced to 62° F., will be found below.

Through the kindness of Professor Rogers, I have been enabled to compare this speculum centimeter with the first centimeter ($\overset{b}{S}_0{}'$) of a standard decimeter upon speculum metal, styled by him $\overset{b}{S}_0$, whose corrections have been definitely established. (See paper on

the "Determinations of the Absolute Lengths of Eight Rowland Gratings," in the Proceedings for 1885.)

I have also compared centimeter "B" with the first centimeter ($\overset{d}{S}_0{}'$) of a standard decimeter styled by Professor Rogers $\overset{d}{S}_0$, which is ruled on the same bar with $\overset{b}{S}_0$.

Having determined the relation of centimeter "B" to each of these centimeters, their respective relative corrections were determined by comparing each of the ten centimeters of each decimeter with the fixed distance between the stops. The results of these various comparisons are as follows:

SERIES (1.)—Comparison of Scale "A" with Scale "B." Objective $=\frac{1}{4}$ inch Bausch & Lomb opaque illuminator. 1 division of micrometer$=0.112\mu$.

1886. Date.	Time of Obs.	Correction to "B."	Date.	Time of Obs.	Correction to "B."
Jan. 12	7.05 A. M.	+0.85μ	Jan. 29	7.05 A. M.	+0.65μ
" 12	7.02 P. M.	+1.31μ	Feb. 28	12.02 P. M.	+0.61μ
" 13	7.06 A. M.	+0.94μ	" 28	0.05 P. M.	+0.68μ
" 15	6.02 P. M.	—0.01μ	Mar. 1	7.00 A. M.	+9.58μ
" 16	7.02 A. M.	+1.12μ	" 2	7.00 A. M.	+0.28μ
" 16	6.00 P. M.	+0.21μ	" 3	7.00 A. M.	—0.19μ
" 17	7.07 A. M.	+1.26μ	" 4	7.00 A. M.	+0.75μ
" 17	9.00 A. M.	+0.33μ	" 5	7.00 A. M.	—0.34μ
" 17	2.08 P. M.	—0.60μ	" 6	7.00 A. M.	+0.02μ
" 17	4.03 P. M.	+0.24μ	" 6	5.00 P. M.	+0.09μ
" 18	7.05 A. M.	+0.38μ	" 7	7.00 A. M.	—0.16μ
" 19	7.06 A. M.	+0.28μ	" 7	1.05 P. M.	+1.83μ
" 20	7.05 A. M.	+0.56μ	" 8	7.00 A. M.	+0.93μ
" 22	7.05 A. M.	+0.77μ	" 9	7.00 A. M.	+1.36μ
" 23	8.05 A. M.	+0.62μ	" 10	7.00 A. M.	—0.07μ
" 24	7.05 A. M.	+0.01μ	" 11	7.00 A. M.	+0.02μ
" 24	9.08 A. M.	+0.28μ	" 12	7.00 A. M.	+0.03μ
" 24	1.20 P. M.	—0.26μ	" 13	7.00 A. M.	+0 03μ
" 24	2.09 P. M.	+0.30μ	" 14	7.00 A. M.	—0.45μ
" 24	4.05 P. M.	+0.65μ	" 14	2.04 P. M.	—0.32μ
" 25	7.05 A. M.	+0.57μ	" 14	5.00 P. M.	+0.09μ
" 25	8.02 A. M.	+0.95μ	" 15	7.00 A. M.	+0.76μ

Jan. 26	7.05 A. M.	$+0.14\mu$	Mar. 16	7.00 A. M.	$+0.55\mu$
" 26	8.02 A. M.	-0.50μ	" 17	7.07 A. M.	$+0.45\mu$
" 27	7.00 A. M.	$+0.18\mu$	" 18	7.00 A. M.	-0.16μ
" 27	9.00 A. M.	$+0.14\mu$	" 19	7.00 A. M.	$+0.69\mu$
" 28	7.08 A. M.	-0.01μ	" 20	7.00 A. M.	$+0.60\mu$
			" 21	7.00 A. M.	$+0.02\mu$

We have, therefore: $B+0.36\mu=A$ (1)

But from the report of Prof. Rogers:

$$B+0.25\mu=\tfrac{1}{100}\,A_0 \quad (2)$$

Hence: $$A-0.11\mu=\tfrac{1}{100}\,A_0 \quad (3)$$

Or, Scale "A" is $\frac{11}{100}\mu$ longer than one-hundredth part of the Meter of the Archives.

SERIES (2).—Comparison of centimeter "B" with the first centimeter of $\overset{b}{S}_0$ ($\overset{b}{S}_0{}'$); objective$=\frac{3}{4}$ inch Bausch & Lomb. 1 division of micrometer$=0.2992\mu$.

The following are the results of the comparisons expressed in divisions of the micrometer:

Date.	Time of Obs.	Correction.	Date.	Time of Obs.	Correction.
June 20	8.00 A. M.	+0.6 div.	June 25	5.06 P. M.	−0.2
" 20	3.00 P. M.	+0.1	" 26	6.00 A. M.	−1.9
" 20	6.03 P. M.	−1.1	" 27	7.05 A. M.	+0.1
" 21	6.03 A. M.	+0.3	" 27	10.02 A. M.	−0.6
" 21	5.03 A. M.	−2.0	" 27	1.02 P. M.	+0.3
" 22	6.05 A. M.	−1.9	" 27	5.05 P. M.	−0.6
" 22	5.00 A. M.	−1.4	" 28	6.05 A. M.	+0.8
" 23	6.00 A. M.	−1.4	" 28	6.07 P. M.	+0.6
" 23	5.00 P. M.	−0.8	" 29	6.02 A. M.	+2.0
" 24	6.05 A. M.	+0.3	" 29	5.05 P. M.	+2.1
" 24	3.02 P. M.	−0.6	" 30	6.05 A. M.	+0.3
" 24	6.05 P. M.	−0.5	" 30	5.08 P. M.	−1.4
" 25	7.02 A. M.	−0.3	July 1	6.02 A. M.	−0.8

Whence, we find: "B"$-0.09\mu=\overset{b}{S}_0{}'$(4)

SERIES (3).—Comparison of centimeter "B" with the first centimeter of $\overset{d}{S}_0{}'$; objective $=\frac{3}{4}$ inch Bausch & Lomb. 1 division of micrometer $=0.2992\mu$.

Date.	Time of Obs.	Correction.	Date	Time of Obs.	Correction.
June 28	6.07 P. M.	+7.7 div.	June 30	5.08 P. M.	+6.6 div.
" 29	5.02 A. M.	+8.6	July 1	6.02 A. M.	+5.7
" 29	5.05 P. M.	+9.4	Aug. 3	7.09 A. M.	+6.9
" 30	6.05 A. M.	+5.9	" 3	4.08 P. M.	+7.9

Whence:

$$B+2.18\mu=\overset{d}{S}_0{}' \ (5).$$

We must now determine the relative corrections of the different centimeters of $\overset{b}{S}_0$ and of $\overset{d}{S}_0$. For $\overset{b}{S}_0$ we have:

Centimeters.	1	2	3	4	5
Comparisons Nos. 1	+0.7 div.	+1.8 div.	—2.5 div.	—3.2 div.	+1.1 div.
" 2	—0.8	+2.6	—4.9	—2.1	+0.4
" 3	+0.7	+3.3	—3.5	—3.4	+1.7
" 4	—0.3	+4.2	—4.6	—4.1	+1.3
" 5	+2.3	+1.9	—4.3	—3.4	+0.5
" 6	+1.9	+3.5	—2.6	—3.6	+0.0
" 7	+2.9	+1.8	—4.8	—3.3	+1.1
" 8	—0.4	+1.3	—2.5	—2.3	—0.2
" 9	+1.5	+3.4	—3.5	—2.6	—0.8
" 10	—1.7	+1.9	—2.6	—2.2	—0.1
" 11	—1.2	+0.7	—4.3	—4.0	—0.3
" 12	—0.8	+2.0	—3.1	—3.0	—0.1
" 13	+0.1	+2.4	—2.1	—2.3	—1.5
" 14	+4.0	+3.6	—3.3	—3.1	—1.5
" 15	—0.7	+2.4	—3.1	—2.9	+0.2
" 16	+0.4	+5.3	—3.5	—2.9	—1.6
" 17	+0.3	+1.8	—2.1	—4.0	+0.7
" 18	+2.0	+3.4	—2.0	—4.1	+1.3
" 19	—0.5	+1.8	—3.7	—4.6	+1.5
" 20	+0.9	+2.1	—2.4	—4.5	+1.0
" 21	—0.1	+2.1	—4.0	—2.5	—1.7
" 22	+0.9	+2.6	—4.4	—3.9	+0.2
" 23	+0.4	+3.6	—3.0	—3.1	—0.1
" 24	+0.2	+2.7	—3.7	—3.9	+0.1
Means reduced to mikrons.	$+0.16\mu$	$+0.78\mu$	-1.00μ	-0.99μ	$+0.04\mu$

Centimeters.		6	7	8	9	10
Comparisons Nos.	1	+2.6 div.	+2.4 div.	—0.3 div	—3.0 div.	+0.6 div.
“	2	+4.5	+3.5	—1.8	—3.0	+2.0
“	3	+2.6	+3.8	—3.0	—2.6	+0.7
“	4	+3.0	+3.8	—1.6	—3.1	+1.3
“	5	+3.1	+4.8	—1.5	—3.0	—0.6
“	6	+1.1	+2.3	—1.6	—2.5	+1.3
“	7	+2.7	+4.8	—2.1	—2.8	—0.1
“	8	+4.4	+2.8	—2.4	—1.5	+0.5
“	9	+3.6	+1.6	—1.3	—2.5	+0.6
“	10	+4.2	+2.3	—1.3	—1.8	+1.1
“	11	+3.0	+2.1	+7.8	—3.3	—0.3
“	12	+3.1	+1.9	—0.6	—2.9	—0.8
“	13	+4.5	+3.4	—2.0	—2.5	+0.2
“	14	+4.1	+2.6	—2.6	—3.0	—1.1
“	15	+4.9	+1.5	—0.6	—2.0	+0.1
“	16	+3.4	+3.4	—2.0	—2.2	—0.2
“	17	+3.6	+3.1	—1.4	—2.2	+0.5
“	18	+2.9	+2.7	—1.3	—5.1	+0.6
“	19	+3.9	+2.7	—0.3	—2.2	+1.8
“	20	+4.7	+3.0	—2.1	—3.0	+0.3
“	21	+3.9	+4.0	—0.6	—1.3	+0.1
“	22	+4.4	+2.8	—0.6	—1.9	+0.2
“	23	+3.9	+2.4	—0.3	—3.3	—0.3
“	24	+3.3	+4.7	—2.0	—2.5	+0.8
Means reduced to mikrons.		+1.08μ	+0.91μ	—0.30μ	—0.80μ	+0.12μ

The following are the relative corrections of the 10 centimeters $\overset{d}{S}_0$:

Comparisons.		1	2	3	4	Means reduced to mikrons.
Centimeters.	1	—5.1 div.	—6.3 div.	—4.0 div.	—5.4 div.	—1.56μ
“	2	+5.6	+7.1	+6.0	+7.2	+1.94
“	3	+1.0	+0.8	—0.5	+0.5	+0.14
“	4	—2.0	—1.9	—0.7	—1.3.	—0.44
“	5	—2.3	—1.8	—2.1	—1.6	—0.58
“	6	—1.0	—1.3	+1.2	—1.5	—0.19
“	7	+2.9	+3.5	+2.9	+3.6	+0.97
“	8	+1.1	+1.3	—0.1	+0.4	+0.20
“	9	—0.3	+0.5	—0.8	—1.0	—0.12
“	10	—0.2	—1.9	—1.6	—1.2	—0.36

From the above comparisons it appears that the relative corrections of the first centimeter of $\overset{b}{S}_0$ and of $\overset{d}{S}_0$ are respectively $+0.16\mu$ and -1.56μ. A full discussion of the corrections for the first five centimeters of $\overset{b}{S}_0$ will be found in the Proceedings of the Society for 1885 (p. 193), from which it appears that the absolute correction for $\overset{b}{S}_0$ is $+0.05\mu$. The absolute correction of $\overset{d}{S}_0$ as communicated to me by Professor Rogers is -1.08μ. The total correction for the first centimeter of $\overset{b}{S}_0$ is therefore $+0.17\mu$, and for the first centimeter of $\overset{d}{S}_0$ is -1.67μ. We have, therefore—

$$\overset{b}{S}_0{}' = \tfrac{1}{100}\,A_0 - 0.17\mu$$
$$\overset{d}{S}_0{}' = \tfrac{1}{100}\,A_0 + 1.67\mu$$

Substituting these values in equations (4) and (5) respectively, we have:

$$\text{"B"} + 0.08\mu = \tfrac{1}{100}\,A_0 \quad (6)$$
$$\text{"B"} + 0.51\mu = \tfrac{1}{100}\,A_0 \quad (7)$$

Giving to equations (6) and (7) the weights 2 and 1 respectively, these numbers being nearly proportional to the square root of the number of observations, we have:

$$\text{"B"} + 0.22\mu = \tfrac{1}{100}\,A_0 \quad (8)$$
$$\text{But} \quad \text{"B"} + 0.36\mu = \text{"A"}$$
$$\text{Hence} \quad \text{"A"} - 0.14\mu = \tfrac{1}{100}\,A_0 \quad (9)$$

I have also compared Scale "A" with the mean of the ten centimeters of each of the decimeters upon glass designated G_4 and G_5. These decimeters were prepared by Professor Rogers and sent to me with the suggestion that Scale "A" should be compared with the mean of the 10 centimeters of the scale in each set of comparisons. In this case the only correction to be applied would be that for total length. Sixteen sets of comparisons were made between "A" and the several centimeters of G_4 and G_5—extending from January 12 to February 4, 1886, giving the following results:

		Correction to "A" in terms of $\frac{1}{10}$ G_4.	Correction to "A" in terms of $\frac{1}{10}$ G_5.
Jan.	12	= + .17μ	— .60μ
"	15	+ .61	+ .30
"	16	— .58	— .24
"	17	— .50	—1.29
"	18	+1.22	+ .41
"	19	—1.00	— .70
"	21	— .53	+ .06
"	23	— .69	— .46
"	25	+ .32	+1.10
"	26	+1.13	+1.30
"	28	+ .81	+ .14
"	29	+ .33	+ .45
"	30	+ .37	+ .95
Feb.	1	+ .63	+ .17
"	2	+ .92	+ .42
"	4	+ .04	+ .26
Means:		+0.21μ	+0.14μ

We have, therefore:

$$\text{"A"} + 0.21\mu = \tfrac{1}{10}\, G_4 \quad (10)$$
$$\text{"A"} + 0.14\mu = \tfrac{1}{10}\, G_5 \quad (11)$$

The absolute corrections required by $\frac{1}{10}$ G_4 and $\frac{1}{10}$ G_5 are unknown to me, Professor Rogers having positively declined to communicate them to me until after this paper had been presented to the Society.

Every comparison given in this paper represents at least eight readings of the micrometer, and often twelve, amounting in all to at least 6000 micrometer readings. Every measurement from January 12, 1886, to the end of the series has been included in making up the final result, and it is hoped that through the multiplication of the comparisons, the accidental errors of observation have been to a large extent eliminated.

NOTE BY PROF. WILLIAM A. ROGERS.

The following are the results of my determinations of the corrections for G_4 and G_5:

For the decimeter G_4 I find:

$$G_4 - 5.71\mu = \tfrac{1}{10}\,A_0.$$

For the decimeter G_5 I find:

$$G_5 - 4.03\mu = \tfrac{1}{10}\,A_0.$$

Hence, for each centimeter we have:

$$\tfrac{1}{10}\,G_4 - 0.57\mu = \tfrac{1}{100}\,A_0.$$
$$\tfrac{1}{10}\,G_5 - 0.40\mu = \tfrac{1}{100}\,A_0.$$

Substituting in Dr. Ewell's equations (10) and (11) we have:

From (10) "A" $- 0.36\mu = \tfrac{1}{100}\,A_0$.
From (11) "A" $- 0.26\mu = \tfrac{1}{100}\,A_0$.

We have, therefore, the following independent determinations of the absolute length of "A" at 62° Fahr., expressed in terms of the *Metre des Archives.*

From equation (3) "A" $- 0.11\mu = \tfrac{1}{100}\,A_0$.
From equation (9) "A" $- 0.14\mu = \tfrac{1}{100}\,A_0$.

From mean of (10) and (11) "A" $- 0.31\mu = \tfrac{1}{100}\,A_0$.

Adopting the mean of these three values we have finally:

"A" $- 0.19\mu = \tfrac{1}{100}\,A_0$.

This result agrees exactly with that found from the bronze bar in the first investigation, and differs only 0.09μ from the final value given in the report.

At the present stage of the investigation we may adopt:

"A" $- 0.17\mu = \tfrac{1}{100}\,A_0$.